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Defect layer method to capture effect of gaps and overlaps in variable stiffness laminates made by Automated Fiber Placement

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ABSTRACT

A variable stiffness design can increase the structural performance of composite laminates. In this paper, a composite laminate with curvilinear fiber paths is designed to maximize simultaneously its in-plane stiffness and buckling load. After obtaining the Pareto front through a surrogate-based optimization algorithm, two variable stiffness laminates among the solution set are selected that can be manufactured by an Automated Fiber Placement machine. Due to the characteristics of the manufacturing process, defects appearing in the form of gaps and/or overlaps emerge within the composite laminate. MATLAB subroutines are developed here to capture the location and extent of the defects. A novel method, called defect layer, is proposed to characterize the change in properties of each layer in the composite laminates that results from the occurrence of gaps and overlaps. Such a method allows calculating the in-plane stiffness and buckling load of a composite laminate with embedded defects. The results show that by incorporating gaps in the laminates the buckling load improvement resulting from fiber steering reduces by 15% compared to the laminates can be observed for a variable stiffness laminate built with a complete overlap strategy.

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1. Introduction

Automated Fiber Placement (AFP) is a technology capable of combining tape placement and filament winding techniques to overcome their limitations and exploit their benefits. The former technique is generally more efficient in manufacturing large and flat panels but its use is limited to components with simple geometry. The latter, on the other hand, has limitations in terms of the manufacturable shape of a final component, which is basically restricted to convex geometries. An AFP machine typically has a self-contained fiber placement head with multiple degrees of freedom (DOF), which is then mounted on a motion base with several translational DOF. A mandrel with an additional rotational DOF provides a tool surface on which a band of tows, called a "course", is placed [1]. As a result, more complex geometries, e.g. concave or double curvature surfaces, can be manufactured. Furthermore, tows can be placed in a pre-designed pattern, e.g. a curvilinear path within the plane of laminates. AFP allows the manufacture of variable stiffness laminates with curvilinear fiber paths which offer a more favorable stress distribution and an improved structural performance [2–6].

It has been demonstrated that variable stiffness laminates can simultaneously maximize buckling load and in-plane stiffness, two conflicting design objectives, as opposed to traditional design strategies of constant stiffness laminates [7,8]. Gürdal et al. [5–7] designed variable stiffness laminates with a curvilinear fiber path, where the fiber angle changes linearly from one end of a plate to the other. They showed that variable stiffness design can decouple the buckling load and the overall in-plane stiffness of the plate. Variable stiffness laminates were designed to provide the same in-plane stiffness as a constant stiffness laminate with higher buckling load and vice versa. Arian Nik et al. [8] used a surrogate-based optimization algorithm and obtained a set of optimum solutions maximizing the in-plane stiffness and buckling load simultaneously. They concluded that both buckling load and inplane stiffness can be increased with respect to a quasi-isotropic laminate. In the previously mentioned works, the minimum turning radius, which determines the maximum amount of steering that is possible with an AFP machine, was not considered in the design process. As a result, not all the solutions obtained could be manufactured via AFP machine. In addition, these previous works assumed no sudden cut tows within the course and defect-free laminates. Due to the manufacturing features inherent to AFP, however, the laminates are not exempt from imperfections; certain defects, mainly gaps and/or overlaps, often appear in the final part, thereby affecting its structural performance [9].

Several authors conducted experiments to investigate the effect of gaps and/or overlaps on the mechanical properties of a constant





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stiffness laminate made by AFP machine. It was found that introducing gaps reduces the laminate strength [10], and the average strain [11], while the overlaps can cause an increase in strength of maximum 13% compared to a non-defective laminate [12], and 93% improvement in buckling load of a panel compared to a straight fiber case [13]. Blom et al. [9] investigated the influence of gaps on the strength and stiffness of variable-stiffness laminates using Finite Element Method (FEM). They found that increasing the total gap area in the laminate deteriorates the strength and stiffness properties. Their work mainly considered gaps and did not model overlaps. In addition, the elements were assumed to be in areas filled with either regular composite material or resin only. This method requires the size of the elements to be small enough to capture precisely the gap areas. Therefore, the number of elements in the FE model drastically increases with the plate size, resulting in a reduced computational efficiency.

In this paper, a novel method, called "defect layer", is introduced to reduce the computational burden of FE analysis of a variable stiffness composite laminate with embedded gaps and/or overlaps. The method enables to calculate precisely gap and overlap area percentage regardless of the number of elements. The paper is organized as follows: Section 2 describes two variable stiffness laminates selected as case studies. In Section 3, location and extent of gaps or overlaps are first determined using MATLAB subroutines developed by the authors [14]. Then, the defect layer method is introduced to build the finite element model of the variable stiffness laminates under investigation. In Section 4, the results, in particular the effect of gaps or overlaps on the in-plane stiffness and the buckling load, are discussed.

2. Problem definition

This section describes the fiber path used to design variable stiffness laminates followed by an explanation of the two test problems including the applied loads and boundary conditions. Two representative designs with an optimum fiber path are selected to investigate the effect of gaps or overlaps on the performance of the laminates.

2.1. Fiber path definition

A variable stiffness laminate can be designed by setting a reference fiber path and offsetting the subsequent fibers to cover the whole laminate. To define the reference fiber path, we consider here a constant curvature path presented by Blom et al. [9]. Along the reference path, the fiber orientation can be obtained as:

$$\sin\varphi = \sin T_0 + \kappa |\mathbf{x}| \tag{1}$$

where φ is the fiber orientation along the fiber path, T_0 is the fiber angle at the plate center, and κ is the curvature of the fiber path. The fiber orientation varies between T_0 (at the plate center, x = 0) and T_1 (at the plate edges, $x = \pm w$) where the curvature of the path remains constant (Fig. 1a). To manufacture the entire plate, the reference fiber path should be shifted along the *y*-direction since the fiber orientation varies along the *x*-direction (Fig. 1b). A single layer with this fiber path definition may be represented by (T_0 , κ), where $\kappa = 0$ represents the case of straight fiber.

2.2. Test problem

We consider a 0.254×0.4064 m (10×16 in) rectangular plate made of 16-ply balanced symmetric laminate subjected to a uniform end shortening along the *y*-direction. Concerning the boundary conditions, the transverse edges are considered free (Fig. 1b) for in-plane displacement and all edges are simply supported against out of plane movement. Carbon epoxy Cytec[®] G40-800/ 5276-1 material properties used in this study are summarized in Table 1.

As representative laminate design for this problem, we select two variable stiffness laminates and we investigate the effect of gaps or overlaps on their in-plane stiffness and buckling load. Variable stiffness laminates are chosen from the set of optimum solutions (Pareto front) obtained by the simultaneous maximization of the in-plane stiffness and buckling load. The Pareto front is obtained using a Non-dominated Sorting Genetic Algorithm-II (NSGAII) integrated with a surrogate model (Radial Basis Function) algorithm (Fig. 2) [8]. The laminate configurations of design (A) and (B), selected here as case studies, are shown in Table 2. Design (A), which offers the maximum achievable buckling load, is chosen to evaluate the real improvement in the buckling load after considering the effect of gaps or overlaps. Design (B), which offers higher buckling load and the same in-plane stiffness compared to the baseline, is chosen to evaluate the effect of gaps or overlaps on both the design objectives.

We note that no gaps or overlaps in variable stiffness laminates are assumed in the calculation of the objective functions, i.e. inplane stiffness and buckling load. The objective functions of the variable stiffness laminates are normalized with respect to the corresponding values of a constant stiffness quasi-isotropic laminate with $[45/0/-45/90]_{2s}$ layup, the baseline.

3. Methodology

In this section, we first explain the approach for locating gaps or overlaps in the selected designs. As an example, the distribution of gaps and overlaps for a lamina in design (A) is illustrated. Then, a defect layer method is proposed to build efficient FE models of composite laminates, which includes gaps or overlaps. Finally, the FE models of the design (A) and (B) are created and the effect of gaps or overlaps on the in-plane stiffness and the buckling load is investigated.

3.1. Identification of gap or overlap locations

To manufacture the selected design laminates, the AFP machine head places the first course along the reference fiber path. Then, the head is offset along the y-direction for placing the subsequent courses to cover the whole laminate. The offset value, i.e. the vertical distance between the left and right course boundaries, is determined to prevent the formation of any major gaps and/or overlaps. As a result, the course width is required to change continuously along the fiber path. In practice, however, the AFP machine can change the course width only by a discrete value via either adding or dropping tows. Thus, small areas of triangular gap and/or overlap appear between adjacent courses. There are several strategies to drop the tows. 0% coverage (complete gap) is a strategy that involves the cutting of a tow as soon as one edge of the tow reaches the course boundary; it creates small triangular areas without fibers, i.e. gaps. The other method is a 100% coverage (complete overlap); here a tow is cut when both edges of the tow cross the course boundary, thereby creating small areas of triangular overlaps. An intermediate scenario is when the coverage is between 0% and 100% [15]. Similar strategies can be followed to add tows, which in turn results in the formation of gaps and/or overlaps.

MATLAB subroutines developed by the authors [14] are used to locate gaps or overlaps for the selected designs. Two strategies, i.e. complete gap and complete overlap are selected to simulate the laminates manufacturing. A complete gap strategy results in a constant thickness laminate, which is essential in certain aerospace applications requiring aerodynamic smoothness. Compared to a



Fig. 1. Reference fiber path; (a) constant curvature fiber path definition. (b) The loading and boundary condition applied to the case studies.

Table 1Material properties.

	G40-800/5276-1	Resin properties	
E_1 (GPa)	143.0	3.7	
<i>E</i> ₂ (GPa)	9.1	3.7	
G (GPa)	4.8	1.4	
v ₁₂	0.3	0.3	



Fig. 2. Pareto front obtained without considering the effect of gaps or overlaps.

complete gap strategy, the complete overlap strategy is often preferred since it provides higher structural improvements, even though the thickness of the final laminate does not remain constant.

We now consider the location of gaps and overlaps in the laminates under investigation in the test problem described in Section 2.2. Fig. 3 shows the location of gaps and overlaps for the [+(44, -1.57)] lamina in design (A). Considering the relative small size of the laminates, eight tows in each course and a tow width of 3.175 mm (1/8 in.) are considered as manufacturing parameters. Fig. 3a shows the location of gaps obtained with a complete gap strategy. The gap area percentage (total gap area divided by the lamina area) is 11.7%. Fig. 3b indicates the location of overlaps for the same lamina using a complete overlap strategy. The total overlap area in this case is 9.5%.

3.2. Defect layer method

Blom et al. [9] have used FE analysis to investigate the effect of gaps on the stiffness and strength of a composite laminate. In their FE model, it is assumed that the elements are completely either in regular composite material or gap areas. As a result of this assumption, the element size was considered to be sufficiently small to capture precisely the gaps. We introduce here a defect layer, which can be a regular composite material with embedded defects (gaps or overlaps). The FE model based on the defect layer method is capable of capturing the effect of gaps or overlaps with a much lower number of elements compared to the existing approach in [9].

The defect layer is similar to a regular composite layer with modified properties or thickness. The defect area percentage (gap or overlap area in each layer of a shell element divided by the element area) is the only parameter used to modify the properties or the thickness of a regular composite layer. It should be noted that for a gap-modified defect layer, elastic properties are reduced, whereas for an overlap-modified defect layer they do not change with respect to those of a regular composite material. While the thickness of a gap-modified defect layer is that of a regular composite layer, the thickness of an overlap-modified defect layer increases proportionally with the overlap area percentage.

Table 2

Selected designs from the Pareto front without considering the effect of gaps or overlaps.

Design	Description	Normalized in-plane stiffness	Normalized buckling load	Layup $[\pm(T_{01}, \kappa_1)/\pm(T_{02}, \kappa_2)/\pm(T_{03}, \kappa_3)/\pm(T_{04}, \kappa_4)]_s$
(A)	Maximum buckling load	0.73	1.37	$[\pm(43, 0.48)/\pm (44, -1.57)/\pm(35, -1.57)/\pm(38, -1.57)]_s$
(B)	Same stiffness and higher buckling load compare to the baseline	1	1.31	$[\pm(43, 0.48)/\pm (48, -1.57)/\pm(30, -1.57)/\pm(26, -1.57)]_s$



Fig. 3. Location of defects in the [+(44, -1.57)] lamina; (a) location of gaps resulting from a complete gap strategy; (b) location of overlaps obtained with a complete overlap strategy.

In a previous work by the authors [16], FE analysis was used to study the effect of gaps or overlaps on the longitudinal compression strength of a quasi-isotropic laminate. The FE models were verified using experimental data. The same approach is used here to build the FE model of the gap-modified defect layer and find the reduction in elastic properties. It should be noted that for the purpose of this study, which is to quantify the effect of defects on in-plane stiffness and buckling load, we focus on the elastic properties only. Further work is necessary to investigate the strength properties of a gap-modified defect layer to perform a strength analysis, such as progressive damage simulation, on variable stiffness panels.

3.2.1. Gap-modified defect layer

A 0.0254×0.0254 m (1 \times 1 in.) single layer $[0]_T$ laminate with a gap at the plate center and along the fiber direction is considered to calculate the properties of the gap-modified defect layer (Fig. 4a).

Longitudinal compression and tension tests along x (fiber direction) and y (transverse direction) axes as well as a shear test have been simulated using FE analysis to find elastic properties (E_X , E_Y , and G) for the gap-modified defect layer. For the sake of brevity, the process is explained here only for calculating E_X . A uniform end-shortening along the fiber direction is applied to the laminate. The laminate is divided into three distinct areas as shown in Fig. 5. Areas 1 and 3 represent non-defective laminate (regular composite material), whereas area 2 denotes the gap area. ANSYS Shell 181 with layerwise formulation, which is a four-node element with six degrees of freedom at each node, is used to mesh the laminate; the element size in the regular and gap areas is identical. The gap width can be varied to change the gap area percentage between 0% (regular composite layer) and 100% (layer completely made of resin). E_X values for the gap-modified defect layer are normalized with respect to the non-defective laminate value, as shown in



Fig. 4. Defect layer; (a) a gap-modified defect layer; (b) an overlap-modified defect layer.

Fig. 6. Elastic properties for the gap-modified defect layer versus gap area percentage are plotted in Fig. 6. These properties can be described as polynomial functions of gap area percentage and then used in FEA.

3.2.2. Overlap-modified defect layer

Fig. 4b shows an overlap-modified defect layer, where a thickness build-up appears along the fiber direction. The elastic properties for the overlap-modified defect layer are the same as those of a regular composite material. Thus, the overlap-modified defect layer can be replaced with a regular composite layer of a higher thickness.

3.3. Building FE model

Once the location of gaps or overlaps have been predicted through the MATLAB subroutines presented in Section 3.1, the FE model of the variable stiffness laminates can be generated in ANSYS. The number of elements should be sufficiently large to avoid altering the panel stiffness distribution, thereby influencing the in-plane stiffness and buckling load.

The local stacking sequence at the mid-point of each element is calculated via Eq. (1) and used in the section property of the multilayer Shell 181 element in ANSYS. We note that each layer of an element might have any defect area percentage; for this purpose MATLAB subroutines are developed to calculate the defect area percentage for each layer of an element.

Fig. 7a shows the real gap distribution for the [+(44, -1.57)] lamina in design (A) plotted earlier in Section 3.1. Fig. 7b illustrates the gap distribution in the FE model obtained with the approach presented in [9]. To generate this model, we used $3.175 \times 3.175 \text{ mm} (1/8 \times 1/8 \text{ in.})$ elements (10240 in total), which might exist completely in composite material areas (white, 0% gap) or in gap areas (black, 100% gap). For unchanged lamina and number of elements, the gap distribution obtained through the defect layer method is depicted in Fig. 7c. We recall here that each layer of an element might have any gap area percentage ranging from white (0% or no gap) to black (100%, complete gap). By comparing the models in Fig. 7b and c, we observe that the latter can capture the geometry, extent and distribution of gaps with higher precision and accuracy than the existing approach in [9].

Furthermore, it is worthy to mention that the element length in the existing approach [9] is governed by the tow width, a model feature that requires a very large number of elements to predict the gap area percentage precisely. However, in the defect layer method, the element size is independent of the tow width, thus a larger element size can be efficiently used to model large structures. For example, the error in calculating the gap area percentage for the [+(44, -1.57)] lamina using the two approaches is shown in Fig. 8. The number of elements is changed between 640 and 40,960. As can be seen in Fig. 8, the error of the existing approach in finding the gap area percentage is random and changes with the number of elements. In contrast, the defect layer method can always predict the exact value of the actual gap area percentage. For plates with overlaps, the process to determine overlap area percentage in each layer of an element is the same as what is explained for gaps; it results in a similar trend as that shown in Fig. 8.

Considering the defect area percentage in each layer of an element, elastic properties can be calculated for the gap-modified layer using Fig. 6, while these properties for the overlap-modified layer are given in Table 1. Defect area percentage is also used to calculate the thickness of an overlap-modified layer while a gapmodified layer has the same thickness as the regular composite layer. Fiber orientation, elastic properties, and the thickness of each layer of an element are passed to ANSYS using ANSYS Parametric Design Language (APDL) codes to create the FE model of a



Fig. 5. Characterization of the elastic properties for the gap-modified defect layer.



Fig. 6. Normalized elastic properties with respect to the gap area percentage for the gap-modified defect layer.

variable stiffness laminate with defects. After building the FE model, the effect of gaps or overlaps on the structural performance of a variable stiffness laminate can be investigated. As an example, we use the FE model obtained by applying the gap-modified defect layer to illustrate (Fig. 9) the gap distribution for design (A), which is a 16-ply laminate. We note that the gap area percentage is averaged through the thickness for each element. 0% in Fig. 9 refers to areas without any gap all through the thickness, and 100% represents areas with a gap all through the thickness.

4. Results and discussion

This section presents the FEA of the variable stiffness laminates, design (A) and (B), incorporating the effect of gaps or overlaps.



Fig. 8. Error in calculating gap area percentage using the approach explained in [9] and the defect layer method.

Fig. 10a shows the real gap distribution in the [+(44, -1.57)] lamina in design (A). The contour of the stress in the *y*-direction for the same lamina under loading and boundary conditions, explained earlier in Section 2.2, is plotted in Fig. 10b. It can be found that the areas with the lowest level of stress have exactly the same shape of gaps depicted in Fig. 10a. Gaps are filled with resin; thus, they can carry a lower amount of load compared to the areas with regular composite material.

Table 3 shows the in-plane stiffness and buckling load of designs (A) and (B) that are normalized with respect to the baseline. As expected, gaps reduce the in-plane stiffness and buckling load for both designs (A) and (B) while overlaps increase both design objectives. Design (A) is expected to provide 37% higher buckling load over the baseline in the case of ignoring the presence of



Fig. 7. Geometry and location of gaps for the [+(44, -1.57)] lamina obtained with: (a) true model; (b) existing approach [9]; (c) defect layer method.



Fig. 9. Gap distribution for design (A).



Fig. 10. (a) The real gap distribution. (b) The contour plot of the stress in the *y*-direction.

manufacturing defects (gaps or overlaps) in the laminate. However, Table 3 shows that with the full gap strategy for manufacturing design (A), there is only 20% improvement in the buckling load compared to the baseline. In other words, the emerging gaps in the laminate (gap area of 12.4%) reduce the buckling load improvement by 12.4%, which is about one third of the expected improvement in the buckling load. On the other hand, the use of the full overlap strategy, which produces thickness build-ups in the laminate (total overlap area of 9.6%), can increase the buckling load by 78% over the baseline. This increase is about two times of the improvement expected when the presence of overlaps is ignored.

For design (A), higher buckling load comes at the cost of a 27% reduction in the in-plane stiffness ignoring the presence of gaps or overlaps. By taking into account the effect of gaps, 15.1% further reduction in the in-plane stiffness is observed. On the other hand, the use of the complete overlap strategy can improve the in-plane stiffness of design (A) by 9.6% compared to the case, where the overlaps are ignored. Generally, it can be concluded that gaps have higher effect on the in-plane stiffness compared to overlaps, while overlaps have higher effect on the buckling load.

Compared to the baseline with no effect of gaps or overlaps, design (B) offers 31% improvement in the buckling load with no change in stiffness. Considering the effect of gaps, the improvement in the buckling load over the baseline is only 15% and it comes with 14% reduction in the in-plane stiffness. As a result, gaps produced during the manufacturing process has the effect of reducing the benefit of fiber steering, whereas overlaps can

Table 3

In-plane stiffness and buckling load after incorporating the effects of gaps or overlaps of designs (A) and (B) normalized with respect to the baseline.

aminate Normalized stiffness		lized ss	Normalized buckling load		
	Value	Change (%)	Value	Change (%)	
Design (A)					
Ignoring defects	0.73	-	1.37	-	
Full gap (total gap area: 12.4%)	0.61	-15.1	1.20	-12.4	
Full overlap (total overlap area: 9.6%)	0.78	+9.6	1.78	+29.9	
Design (B)					
Ignoring defects	1	-	1.31	-	
Full gap (total gap area: 12.3%)	0.86	-14	1.15	-12.2	
Full overlap (total overlap area: 9.4%)	1.11	+11	1.71	+30.5	

increase the in-plane stiffness and the buckling load over the baseline by 11% and 71%, respectively.

5. Conclusion

In this work, the effect of gaps or overlaps on the in-plane stiffness and buckling load of variable stiffness laminates has been investigated. A defect layer method has been introduced to capture the geometry and location of gaps and overlaps, and applied to two laminate designs obtained from the optimal solutions of the Pareto front. Compared with the outcome obtained with an alternative approach existing in literature, the results demonstrate that the defect layer method is more precise in locating and calculating the defect area percentage, regardless of the number of elements.

With reference to the two variable stiffness laminates of the test problem investigated in this paper, the following results have been observed: gaps deteriorate both in-plane stiffness and buckling load, whereas overlaps improve the structural performance. In particular, for the first laminate configuration, the improvement in buckling load resulting from the fiber steering (37%) decreases to 20% when the effect of gaps is modelled. On the other hand, overlaps increase the improvement in the buckling load to 78%. For the second laminate configuration, overlaps have been shown to improve the in-plane stiffness and buckling load by 11% and 71%, respectively. It should be noted that the gains in the in-plane stiffness and buckling load may depend on the loading and boundary conditions.

Future work is required to obtain a Pareto front that considers the effect of gaps or overlaps on the in-plane stiffness and the buckling load of the laminates. These results would provide important design guidelines of direct interest to industry. The effect of manufacturing parameters, e.g. tow width and number of tows in one course, which affect defect size and distribution, can also be investigated. Different loading and boundary conditions can also be considered. Furthermore, strength properties for a defect layer can be derived and used in a progressive damage simulation of variable stiffness laminates with gaps and/or overlap.

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